Let

We will prove two lemmas first.

|  |  |
| --- | --- |
|  |  |
|  |  |
| from the definition of |  |
| Distributive property |  |
| since it’s a mistake |  |
| from definition of and |  |
|  |  |
| By induction |  |

The last step is proved by induction:

|  |  |
| --- | --- |
| or | |
|  |  |
| from the definition of |  |
|  |  |
|  | |
|  |  |
| from |  | |
| from the induction hypothesis |  | |
|  |  | |

|  |  |
| --- | --- |
|  |  |
|  |  |
| from definition of |  |
| distributive property |  |
| from the definition of |  |
| by induction |  |

The last step follows from another induction proof:

|  |  |
| --- | --- |
|  | |
|  |  |
| from the definition of |  |
|  |  |
|  | |
| from the definition of |  |
| distributive property |  |
| The first two terms is the LHS of the induction hypothesis and the last term is bounded by . |  |
|  | = |
|  |  |

Now that we proved the two lemmas above, we can use them to arrive at the conclusion:

|  |  |
| --- | --- |
| From lemma 2 |  |
| by definition; |  |
|  |  |
| From lemma 1 |  |

It follows from that and we can conclude that the number of iterations is bounded by .

**Reference:**

1. Professor Ross’ Slides on Perceptron Algorithm
2. Andrew Ng’s CS229 lecture note